GLIMMPSE Validation Report:

GLMM(F) Example 5. Power for a test of interaction in a multivariate model

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1. Introduction

The following report contains validation results for the JavaStatistics library, a component of the GLIMMPSE software system. For more information about GLIMMPSE and related publications, please visit

http://samplesizeshop.org.

The automated validation tests shown below compare power values produced by the JavaStatistics library to published results and also to simulation. Sources for published values include POWERLIB (Johnson *et al.* 2007) and a SAS IML implementation of the methods described by Glueck and Muller (2003).

Validation results are listed in Section 3 of the report. Timing results show the calculation and simulation times for the overall experiment and the mean times per power calculation. Summary statistics show the maximum absolute deviation between the power value calculated by the JavaStatistics library and the results obtained from SAS or via simulation. The table in Section 3.3 shows the deviation values for each individual power comparison. Deviations larger than 10^{-6} from SAS power values and 0.05 for simulated power values are displayed in red.

2. Study Design

The study design for Example 5 is a balanced four-sample designwith three repeated measures over time. We calculate power for a test of the group by time interaction. The unstructured covariance model is most appropriate for the design. The example demonstrates the difference in power depending on the choice of statistical test when assumptions of sphericity are unlikely to hold.

2.1. Inputs to the Power Calculation

2.1.1. List Inputs

Type I error rates

0.0100000

Beta scale values

0.0000000, 0.5000000, 1.0000000, 1.5000000, 2.0000000

Sigma scale values

1.0000000, 2.0000000

Per group sample size values

5, 10

Statistical tests

UNIREP, UNIREP-BOX, UNIREP-GG, UNIREP-HF, WL, PBT, HLT

Power methods

cond

2.1.2. Matrix Inputs

$$\underset{(4\times4)}{\text{Es}\,(\mathbf{X})} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

$$\underset{(4\times3)}{\mathbf{B}} = \begin{bmatrix} 2.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

$$\underset{(3\times4)}{\mathbf{C}} = \begin{bmatrix} 1.0000 & -1.0000 & 0.0000 & 0.0000 \\ 1.0000 & 0.0000 & -1.0000 & 0.0000 \\ 1.0000 & 0.0000 & 0.0000 & -1.0000 \end{bmatrix}$$

$$\underset{(3\times2)}{\mathbf{U}} = \begin{bmatrix} 1.0000 & 1.0000 \\ -1.0000 & 0.0000 \\ 0.0000 & -1.0000 \end{bmatrix}$$

$$\underset{(3\times2)}{\mathbf{\Theta}_0} = \begin{bmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.0000 \\ 0.0000 & 0.0000 \end{bmatrix}$$

$$\underset{(3\times3)}{\mathbf{\Sigma}_E} = \begin{bmatrix} 1.0000 & 0.4000 & 0.4000 \\ 0.4000 & 1.0000 & 0.4000 \\ 0.4000 & 0.4000 & 1.0000 \end{bmatrix}$$

3. Validation Results

A total of 140 power values were computed for this experiment.

3.1. Timing

	Total Time (seconds)	Mean Time (seconds)
Calculation	0.0160000	1.14E-4
Simulation	45.0830000	3.22E-1

3.2. Summary Statistics

Max deviation from SAS	0.00000097
Max deviation from simulation	0.07025271

3.3. Full Validation Results

Power	SAS Power (deviation)	Sim Power (deviation)	Test	Sigma Scale	Beta Scale	Total N	Alpha
0.0100000	0.0100000 (0.0000000)	0.0093000 (0.0007000)	UNIREP	1.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0107000 (0.0007000)	UNIREP	1.0000000	0.0000000	40	0.0100000
0.0211588	0.0211581 (0.0000007)	0.0200000 (0.0011588)	UNIREP	1.0000000	0.5000000	20	0.0100000
0.0418834	0.0418830 (0.0000004)	0.0426000 (0.0007166)	UNIREP	1.0000000	0.5000000	40	0.0100000
0.0813945	0.0813942 (0.0000003)	0.0816000 (0.0002055)	UNIREP	1.0000000	1.0000000	20	0.0100000
0.2662637	0.2662633 (0.0000004)	0.2625000 (0.0037637)	UNIREP	1.0000000	1.0000000	40	0.0100000
0.2592467	0.2592465 (0.0000002)	0.2569000 (0.0023467)	UNIREP	1.0000000	1.5000000	20	0.0100000
0.7268313	0.7268312 (0.0000002)	0.7180000 (0.0088313)	UNIREP	1.0000000	1.5000000	40	0.0100000
0.5601981	0.5601976 (0.0000005)	0.5569000 (0.0032981)	UNIREP	1.0000000	2.0000000	20	0.0100000
0.9697551	0.9697548 (0.0000002)	0.9704000 (0.0006449)	UNIREP	1.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0093000 (0.0007000)	UNIREP	2.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0107000 (0.0007000)	UNIREP	2.0000000	0.0000000	40	0.0100000
0.0150475	0.0150473 (0.0000002)	0.0135000 (0.0015475)	UNIREP	2.0000000	0.5000000	20	0.0100000
0.0230475	0.0230470 (0.0000006)	0.0229000 (0.0001475)	UNIREP	2.0000000	0.5000000	40	0.0100000

0.0367117	0.0367111 (0.0000005)	0.0337000 (0.0030117)	UNIREP	2.0000000	1.0000000	20	0.0100000
0.0973219	0.0973218 (0.0000001)	0.1000000 (0.0026781)	UNIREP	2.0000000	1.0000000	40	0.0100000
0.0953062	0.0953061 (0.0000001)	0.0947000 (0.0006062)	UNIREP	2.0000000	1.5000000	20	0.0100000
0.3158718	0.3158716 (0.0000002)	0.3144000 (0.0014718)	UNIREP	2.0000000	1.5000000	40	0.0100000
0.2180674	0.2180672 (0.0000003)	0.2163000 (0.0017674)	UNIREP	2.0000000	2.0000000	20	0.0100000
0.6511105	0.6511103 (0.0000002)	0.6424000 (0.0087105)	UNIREP	2.0000000	2.0000000	40	0.0100000
0.0006932	0.0006932 (0.0000000)	0.0009000 (0.0002068)	UNIREP-BOX	1.0000000	0.0000000	20	0.0100000
0.0007996	0.0007996 (0.0000000)	0.0009000 (0.0001004)	UNIREP-BOX	1.0000000	0.0000000	40	0.0100000
0.0018873	0.0018873 (0.0000001)	0.0015000 (0.0003873)	UNIREP-BOX	1.0000000	0.5000000	20	0.0100000
0.0056223	0.0056213 (0.0000010)	0.0050000 (0.0006223)	UNIREP-BOX	1.0000000	0.5000000	40	0.0100000
0.0114946	0.0114944 (0.0000002)	0.0098000 (0.0016946)	UNIREP-BOX	1.0000000	1.0000000	20	0.0100000
0.0762645	0.0762637 (0.0000008)	0.0803000 (0.0040355)	UNIREP-BOX	1.0000000	1.0000000	40	0.0100000
0.0604959	0.0604955 (0.0000004)	0.0599000 (0.0005959)	UNIREP-BOX	1.0000000	1.5000000	20	0.0100000
0.4132896	0.4132891 (0.0000005)	0.4126000 (0.0006896)	UNIREP-BOX	1.0000000	1.5000000	40	0.0100000
0.2151713	0.2151711 (0.0000003)	0.2122000 (0.0029713)	UNIREP-BOX	1.0000000	2.0000000	20	0.0100000
0.8490847	0.8490843 (0.0000005)	0.8481000 (0.0009847)	UNIREP-BOX	1.0000000	2.0000000	40	0.0100000
0.0006932	0.0006932 (0.0000000)	0.0009000 (0.0002068)	UNIREP-BOX	2.0000000	0.0000000	20	0.0100000
0.0007996	0.0007996 (0.0000000)	0.0009000 (0.0001004)	UNIREP-BOX	2.0000000	0.0000000	40	0.0100000
0.0012002	0.0011999 (0.0000003)	0.0009000 (0.0003002)	UNIREP-BOX	2.0000000	0.5000000	20	0.0100000
0.0025068	0.0025067 (0.0000001)	0.0025000 (0.0000068)	UNIREP-BOX	2.0000000	0.5000000	40	0.0100000
0.0039240	0.0039239 (0.0000001)	0.0039000 (0.0000240)	UNIREP-BOX	2.0000000	1.0000000	20	0.0100000
0.0178607	0.0178600 (0.0000007)	0.0160000 (0.0018607)	UNIREP-BOX	2.0000000	1.0000000	40	0.0100000

0.0142876	0.0142870 (0.0000006)	0.0138000 (0.0004876)	UNIREP-BOX	2.0000000	1.5000000	20	0.0100000
0.0990176	0.0990170 (0.0000006)	0.0998000 (0.0007824)	UNIREP-BOX	2.0000000	1.5000000	40	0.0100000
0.0466399	0.0466395 (0.0000004)	0.0474000 (0.0007601)	UNIREP-BOX	2.0000000	2.0000000	20	0.0100000
0.3333343	0.3333339 (0.0000004)	0.3285000 (0.0048343)	UNIREP-BOX	2.0000000	2.0000000	40	0.0100000
0.0072004	0.0072004 (0.0000000)	0.0068000 (0.0004004)	UNIREP-GG	1.0000000	0.0000000	20	0.0100000
0.0086403	0.0086403 (0.0000000)	0.0095000 (0.0008597)	UNIREP-GG	1.0000000	0.0000000	40	0.0100000
0.0157779	0.0157771 (0.0000008)	0.0151000 (0.0006779)	UNIREP-GG	1.0000000	0.5000000	20	0.0100000
0.0373965	0.0373961 (0.0000004)	0.0389000 (0.0015035)	UNIREP-GG	1.0000000	0.5000000	40	0.0100000
0.0647193	0.0647189 (0.0000004)	0.0662000 (0.0014807)	UNIREP-GG	1.0000000	1.0000000	20	0.0100000
0.2494066	0.2494061 (0.0000005)	0.2469000 (0.0025066)	UNIREP-GG	1.0000000	1.0000000	40	0.0100000
0.2208229	0.2208226 (0.0000003)	0.2206000 (0.0002229)	UNIREP-GG	1.0000000	1.5000000	20	0.0100000
0.7089298	0.7089296 (0.0000002)	0.7000000 (0.0089298)	UNIREP-GG	1.0000000	1.5000000	40	0.0100000
0.5093195	0.5093193 (0.0000002)	0.5089000 (0.0004195)	UNIREP-GG	1.0000000	2.0000000	20	0.0100000
0.9658454	0.9658450 (0.0000004)	0.9668000 (0.0009546)	UNIREP-GG	1.0000000	2.0000000	40	0.0100000
0.0072004	0.0072004 (0.0000000)	0.0068000 (0.0004004)	UNIREP-GG	2.0000000	0.0000000	20	0.0100000
0.0086403	0.0086403 (0.0000000)	0.0095000 (0.0008597)	UNIREP-GG	2.0000000	0.0000000	40	0.0100000
0.0110465	0.0110463 (0.0000002)	0.0108000 (0.0002465)	UNIREP-GG	2.0000000	0.5000000	20	0.0100000
0.0203037	0.0203031 (0.0000006)	0.0208000 (0.0004963)	UNIREP-GG	2.0000000	0.5000000	40	0.0100000
0.0280776	0.0280770 (0.0000006)	0.0274000 (0.0006776)	UNIREP-GG	2.0000000	1.0000000	20	0.0100000
0.0886478	0.0886477 (0.0000002)	0.0917000 (0.0030522)	UNIREP-GG	2.0000000	1.0000000	40	0.0100000
0.0764113	0.0764111 (0.0000002)	0.0782000 (0.0017887)	UNIREP-GG	2.0000000	1.5000000	20	0.0100000
0.2975229	0.2975226 (0.0000003)	0.2958000 (0.0017229)	UNIREP-GG	2.0000000	1.5000000	40	0.0100000

0.1835721	0.1835718 (0.0000004)	0.1844000 (0.0008279)	UNIREP-GG	2.0000000	2.0000000	20	0.0100000
0.6313763	0.6313761 (0.0000003)	0.6238000 (0.0075763)	UNIREP-GG	2.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0068000 (0.0032000)	UNIREP-HF	1.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0095000 (0.0005000)	UNIREP-HF	1.0000000	0.0000000	40	0.0100000
0.0211588	0.0211581 (0.0000007)	0.0151000 (0.0060588)	UNIREP-HF	1.0000000	0.5000000	20	0.0100000
0.0418834	0.0418830 (0.0000004)	0.0389000 (0.0029834)	UNIREP-HF	1.0000000	0.5000000	40	0.0100000
0.0813945	0.0813942 (0.0000003)	0.0662000 (0.0151945)	UNIREP-HF	1.0000000	1.0000000	20	0.0100000
0.2662637	0.2662633 (0.0000004)	0.2469000 (0.0193637)	UNIREP-HF	1.0000000	1.0000000	40	0.0100000
0.2592467	0.2592465 (0.0000002)	0.2206000 (0.0386467)	UNIREP-HF	1.0000000	1.5000000	20	0.0100000
0.7268313	0.7268312 (0.0000002)	0.7000000 (0.0268313)	UNIREP-HF	1.0000000	1.5000000	40	0.0100000
0.5601981	0.5601976 (0.0000005)	0.5089000 (0.0512981)	UNIREP-HF	1.0000000	2.0000000	20	0.0100000
0.9697551	0.9697548 (0.0000002)	0.9668000 (0.0029551)	UNIREP-HF	1.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0068000 (0.0032000)	UNIREP-HF	2.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0095000 (0.0005000)	UNIREP-HF	2.0000000	0.0000000	40	0.0100000
0.0150475	0.0150473 (0.0000002)	0.0108000 (0.0042475)	UNIREP-HF	2.0000000	0.5000000	20	0.0100000
0.0230475	0.0230470 (0.0000006)	0.0208000 (0.0022475)	UNIREP-HF	2.0000000	0.5000000	40	0.0100000
0.0367117	0.0367111 (0.0000005)	0.0274000 (0.0093117)	UNIREP-HF	2.0000000	1.0000000	20	0.0100000
0.0973219	0.0973218 (0.0000001)	0.0917000 (0.0056219)	UNIREP-HF	2.0000000	1.0000000	40	0.0100000
0.0953062	0.0953061 (0.0000001)	0.0782000 (0.0171062)	UNIREP-HF	2.0000000	1.5000000	20	0.0100000
0.3158718	0.3158716 (0.0000002)	0.2958000 (0.0200718)	UNIREP-HF	2.0000000	1.5000000	40	0.0100000
0.2180674	0.2180672 (0.0000003)	0.1844000 (0.0336674)	UNIREP-HF	2.0000000	2.0000000	20	0.0100000
0.6511105	0.6511103 (0.0000002)	0.6238000 (0.0273105)	UNIREP-HF	2.0000000	2.0000000	40	0.0100000

0.0100000	0.0100000 (0.0000000)	0.0090000 (0.0010000)	WL	1.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0089000 (0.0011000)	WL	1.0000000	0.0000000	40	0.0100000
0.0207978	0.0207971 (0.0000007)	0.0180000 (0.0027978)	WL	1.0000000	0.5000000	20	0.0100000
0.0411985	0.0411981 (0.0000003)	0.0382000 (0.0029985)	WL	1.0000000	0.5000000	40	0.0100000
0.0749141	0.0749139 (0.0000002)	0.0661000 (0.0088141)	WL	1.0000000	1.0000000	20	0.0100000
0.2468162	0.2468160 (0.0000002)	0.2333000 (0.0135162)	WL	1.0000000	1.0000000	40	0.0100000
0.2184787	0.2184783 (0.0000004)	0.1889000 (0.0295787)	WL	1.0000000	1.5000000	20	0.0100000
0.6601275	0.6601272 (0.0000002)	0.6555000 (0.0046275)	WL	1.0000000	1.5000000	40	0.0100000
0.4501706	0.4501703 (0.0000003)	0.4211000 (0.0290706)	WL	1.0000000	2.0000000	20	0.0100000
0.9288726	0.9288724 (0.0000002)	0.9455000 (0.0166274)	WL	1.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0090000 (0.0010000)	WL	2.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0089000 (0.0011000)	WL	2.0000000	0.0000000	40	0.0100000
0.0149309	0.0149307 (0.0000002)	0.0126000 (0.0023309)	WL	2.0000000	0.5000000	20	0.0100000
0.0228970	0.0228964 (0.0000005)	0.0200000 (0.0028970)	WL	2.0000000	0.5000000	40	0.0100000
0.0353249	0.0353244 (0.0000005)	0.0302000 (0.0051249)	WL	2.0000000	1.0000000	20	0.0100000
0.0936362	0.0936361 (0.0000001)	0.0872000 (0.0064362)	WL	2.0000000	1.0000000	40	0.0100000
0.0867877	0.0867871 (0.0000006)	0.0752000 (0.0115877)	WL	2.0000000	1.5000000	20	0.0100000
0.2908500	0.2908492 (0.0000008)	0.2767000 (0.0141500)	WL	2.0000000	1.5000000	40	0.0100000
0.1863589	0.1863583 (0.0000006)	0.1596000 (0.0267589)	WL	2.0000000	2.0000000	20	0.0100000
0.5895650	0.5895646 (0.0000004)	0.5796000 (0.0099650)	WL	2.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0081000 (0.0019000)	PBT	1.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0082000 (0.0018000)	PBT	1.0000000	0.0000000	40	0.0100000

0.0215038	0.0215037 (0.0000001)	0.0151000 (0.0064038)	PBT	1.0000000	0.5000000	20	0.0100000
0.0418314	0.0418310 (0.0000005)	0.0348000 (0.0070314)	PBT	1.0000000	0.5000000	40	0.0100000
0.0758690	0.0758687 (0.0000003)	0.0514000 (0.0244690)	PBT	1.0000000	1.0000000	20	0.0100000
0.2381883	0.2381881 (0.0000002)	0.2151000 (0.0230883)	PBT	1.0000000	1.0000000	40	0.0100000
0.1987074	0.1987072 (0.0000002)	0.1457000 (0.0530074)	PBT	1.0000000	1.5000000	20	0.0100000
0.6012579	0.6012574 (0.0000005)	0.6148000 (0.0135421)	PBT	1.0000000	1.5000000	40	0.0100000
0.3637696	0.3637694 (0.0000002)	0.3089000 (0.0548696)	PBT	1.0000000	2.0000000	20	0.0100000
0.8588473	0.8588471 (0.0000002)	0.9291000 (0.0702527)	PBT	1.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0081000 (0.0019000)	PBT	2.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0082000 (0.0018000)	PBT	2.0000000	0.0000000	40	0.0100000
0.0152650	0.0152646 (0.0000004)	0.0120000 (0.0032650)	PBT	2.0000000	0.5000000	20	0.0100000
0.0232160	0.0232153 (0.0000007)	0.0188000 (0.0044160)	PBT	2.0000000	0.5000000	40	0.0100000
0.0366875	0.0366866 (0.0000008)	0.0261000 (0.0105875)	PBT	2.0000000	1.0000000	20	0.0100000
0.0939413	0.0939412 (0.0000001)	0.0796000 (0.0143413)	PBT	2.0000000	1.0000000	40	0.0100000
0.0870532	0.0870524 (0.0000008)	0.0589000 (0.0281532)	PBT	2.0000000	1.5000000	20	0.0100000
0.2780905	0.2780898 (0.0000007)	0.2535000 (0.0245905)	PBT	2.0000000	1.5000000	40	0.0100000
0.1731813	0.1731809 (0.0000004)	0.1228000 (0.0503813)	PBT	2.0000000	2.0000000	20	0.0100000
0.5398027	0.5398026 (0.0000002)	0.5405000 (0.0006973)	PBT	2.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0084000 (0.0016000)	HLT	1.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0095000 (0.0005000)	HLT	1.0000000	0.0000000	40	0.0100000
0.0192582	0.0192573 (0.0000008)	0.0184000 (0.0008582)	HLT	1.0000000	0.5000000	20	0.0100000
0.0392084	0.0392079 (0.0000004)	0.0384000 (0.0008084)	HLT	1.0000000	0.5000000	40	0.0100000

0.0659640	0.0659634 (0.0000006)	0.0673000 (0.0013360)	HLT	1.0000000	1.0000000	20	0.0100000
0.2415354	0.2415346 (0.0000008)	0.2386000 (0.0029354)	HLT	1.0000000	1.0000000	40	0.0100000
0.2015113	0.2015106 (0.0000007)	0.1947000 (0.0068113)	HLT	1.0000000	1.5000000	20	0.0100000
0.6833024	0.6833017 (0.0000007)	0.6733000 (0.0100024)	HLT	1.0000000	1.5000000	40	0.0100000
0.4506044	0.4506041 (0.0000003)	0.4428000 (0.0078044)	HLT	1.0000000	2.0000000	20	0.0100000
0.9553919	0.9553917 (0.0000002)	0.9531000 (0.0022919)	HLT	1.0000000	2.0000000	40	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0084000 (0.0016000)	HLT	2.0000000	0.0000000	20	0.0100000
0.0100000	0.0100000 (0.0000000)	0.0095000 (0.0005000)	HLT	2.0000000	0.0000000	40	0.0100000
0.0142500	0.0142497 (0.0000003)	0.0130000 (0.0012500)	HLT	2.0000000	0.5000000	20	0.0100000
0.0220754	0.0220748 (0.0000006)	0.0191000 (0.0029754)	HLT	2.0000000	0.5000000	40	0.0100000
0.0316271	0.0316264 (0.0000008)	0.0313000 (0.0003271)	HLT	2.0000000	1.0000000	20	0.0100000
0.0890653	0.0890651 (0.0000002)	0.0892000 (0.0001347)	HLT	2.0000000	1.0000000	40	0.0100000
0.0765132	0.0765129 (0.0000003)	0.0757000 (0.0008132)	HLT	2.0000000	1.5000000	20	0.0100000
0.2869464	0.2869459 (0.0000005)	0.2846000 (0.0023464)	HLT	2.0000000	1.5000000	40	0.0100000
0.1697335	0.1697326 (0.0000008)	0.1657000 (0.0040335)	HLT	2.0000000	2.0000000	20	0.0100000
0.6066502	0.6066495 (0.0000007)	0.5927000 (0.0139502)	HLT	2.0000000	2.0000000	40	0.0100000

References

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